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A Simple Heuristic Algorithm to Solve the Bulk Transportation Problem

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Abstract— A Bulk Transportation Problem(BTP) deals with the problem of minimizing the total bulk transportation cost. It differs from the Classical transportation problem in the sense that the total requirement of each destination is to be satisfied from only one source; however subject to the availability of the product at the source, a source can supply to any number of destinations. In this paper, the minimum cost of BTP is obtained by a heuristic method hence providing a simple and alternative procedure to obtain the minimum cost of the BTP.

Index Terms— *Transportation Problem, Bulk Transportation, Heuristic Method.*

1 Introduction

THE Classical transportation problem is a subclass of linear programming problem, which has been studied extensively in literature. A large number of methods have been developed for solving the Classical transportation problem. The Classical transportation problem was presented by Hitchcock [1]. Dantzig [2] further developed the theory of Classical transportation problem. Several authors [3],[4],[5],[6],[7],[8],[9],[10],[11],[12] studied different single objective transportation problems. The BTP is a special class of transportation problems introduced in literature by Maio and Roveda [13] with the objective of minimizing the total bulk transportation cost. The authors solved the problem by an iterative procedure. The authors also gave an industrial application of the BTP wherein different warehouses of a firm are supplying to different shops; each shop was supplied from only one warehouse to maintain organizational efficiency. Later on, an algorithm based on the branch and bound method was presented by Srinivasan and Thompson [14]. A method based on lexicographic minimum to solve the BTP was developed by Murthy [15]. Bhatia [16], Foulds and Gibbons [17] discussed the cost minimizing BTP. Verma and Puri [18] proposed a branch and bound method for cost minimizing BTP. The present paper presents a much simpler and alternative solution procedure for the BTP, the application of which is very simple as compared to the existing methods. In Section 2, the formulation of the BTP is given. Section 3 discusses the steps of the proposed algorithm. In Section 4, a numerical example is considered. Section 5 gives a comparative study of the proposed method with existing methods [13],[14]. Lastly in Section 6, some concluding remarks are presented.

2 Formulation of the Problem

Let there be 'm' sources (S_i) producing a particular product and 'n' destinations (D_j) having some requirement. Let C denote the total cost of bulk transportation.
The mathematical formulation of the problem is as follows

Minimize $$\boldsymbol{C} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij} \qquad (1)$$

subject to the constraints

$$\sum_{j=1}^{n} b_j x_{ij} \le a_i \ (i = 1,2,3, \dots\dots.., m) \qquad (2)$$

$$\sum_{i=1}^{m} x_{ij} = 1 \ (j = 1,2,3, \dots\dots.., n) \qquad (3)$$

$$x_{ij} = 0 \ or\ 1 \ (i = 1,2,3,..,m;\ j = 1,2,3,..,n) \qquad (4)$$

where a_i, b_j *and* c_{ij} are non-negative real numbers defined below:
a_i is the number of units of a product available at the ith source .
b_j is the number of units of a product required at jth destination.
c_{ij} is the cost of bulk transportation of product from ith source to jth destination.
x_{ij} is the decision variables assuming the value 1 or 0 depending upon whether the demand at the destination j is met or not met from the source *i*.

3 Proposed Algorithm

Steps of the proposed algorithm:
Step 3.1
Delete cells (i, j) from the initial table for which availability a_i is less than requirement b_j.
Step 3.2
Select the two smallest bulk transportation costs for each row and column and find their difference. This difference indicates the penalty. This penalty indicates an extra cost which has to be paid if the cell having minimum bulk transportation cost remains unallocated.
Step 3.3
a) Select the maximum penalty corresponding to each row i and each column j and identify the least cost cell (i, j) and allocate 1 to this cell (i, j).This means that requirement at destination j will be met from source i. In case of tie among the penalties, select the cell (i, j) where maximum allocation is possible in a selected row or column.

b) Reduce the availability of source i by the requirement of destination j whose requirement has been met.

Step 3.4

Remove the rows or columns from the table having zero availability or zero requirement and repeat steps 1 to 3 until the requirements of all destinations is satisfied.

4 Numerical Problem

The numerical problem by Maio & Roveda [13] is considered here and the proposed algorithm is applied to the problem. The tableau representation of numerical problem is given in Table 1.

Table 1(Representation of Costs of BTP)

	D1	D2	D3	D4	D5	$a_i \downarrow$
S1	2	3	4	7	1	5
S2	4	1	1	8	8	4
S3	1	7	11	1	6	3
S4	-	-	10	3	5	2
$bj \rightarrow$	3	3	2	2	1	

After applying Step 1 of the algorithm, we get table 1 as cells (4,1) and (4,2) have been deleted ,a_4 being smaller than b_1 and b_2. It is to be noted here that Cells (4,1) and (4,2) in Table 1 above are given vacant in [13] due to x_{41} and x_{42} being zero.

Table 2(Reduced table after step 1)

	D1	D2	D3	D4	D5	$a_i \downarrow$
S1	2	3	4	7	1	5
S2	4	1	1	8	8	4
S3	1	7	11	1	6	3
S4	-	-	10	3	5	2
$bj \rightarrow$	3	3	2	2	1	

Next, we apply step 2 and calculate the difference between the least two bulk transportation costs for each row and column. The penalties for sources S_1, S_2, S_3 and S_4 are 1, 0, 0 and 2 respectively and the penalties for destinations D_1, D_2, D_3, D_4 and D_5 are 1, 2, 3, 2 and 4. Out of these penalties, the maximum penalty is 4 which is associated with the destination D_5. Minimum cost in the column corresponding to destination D_5 is 1 in the cell (1,5) , so allocate in the cell (1,5) $i.e.\ x_{15} = 1$. This satisfies the requirement of destination D_5 . Remove the destination D_5 from the table and update the table to obtain Table 3.

Table 3(Reduced table after 1st allocation)

	D1	D2	D3	D4	$a_i \downarrow$
S1	2	3	4	7	4
S2	4	1	1	8	4
S3	1	7	11	1	3
S4	-	-	10	3	2
$bj \rightarrow$	3	3	2	2	

In table 3, the penalties for sources S_1, S_2, S_3 and S_4 are 1, 0, 0 and 7 respectively and for destinations D_1, D_2, D_3 *and* D_4 are 1, 2, 3 and 2 respectively. Out of these penalties, the maximum penalty is 7 corresponding to the source S_4. Minimum cost in the row corresponding to source S_4 is 3 in the cell (4,4), so allocate in the cell (4,4) $i.e.\ x_{44} = 1$. This satisfies the requirement of destination D_4 and this also exhausts the availability of source S_4. Remove the source S_4 and destination D_4 from the table and update the table to obtain table 4.

Table 4(Reduced table after 2nd allocation)

	D1	D2	D3	$a_i \downarrow$
S1	2	3	4	4
S2	4	1	1	4
S3	1	7	11	3
$b_j \rightarrow$	3	3	2	

In table 4, the penalties for sources S_1, S_2 *and* S_3 are 1, 0 and 6 respectively and for destinations D_1, D_2 *and* D_3 are 1, 2 and 3 respectively. Out of these penalties, the maximum penalty is 6 corresponding to the source S_3. Minimum cost in the row corresponding to source S_3 is 1 in the cell(3,1), so allocate in the cell (3,1) $i.e.\ x_{31} = 1$. This satisfies the requirement of destination D_1 and this also exhausts the availability of the source S_3. Remove the destination D_1 and source S_3 from the table and update the table to obtain table 5.

Table 5(Reduced table after 3rd allocation)

	D2	D3	$a_i \downarrow$
S1	3	4	4
S2	1	1	4
$b_j \rightarrow$	3	2	

In table 5, the penalties for sources S_1 *and* S_2 are 1 and 0 respectively and for destinations D_2 *and* D_3 are 2 and 3 respectively. Out of these penalties, the maximum penalty is 3 corresponding to the destination D_3. Minimum cost in the column corresponding to destination D_3 is 1 in the cell (2,3), so allocate in the cell (2,3) $i.e.\ x_{23} = 1$. This satisfies the requirement of destination D_3. Remove the destination D_3 from the table and update the table to obtain table 6.

Table 6(Reduced table after 4th allocation)

	D_2	$a_i \downarrow$
S_1	3	4
S_2	-	2
$b_j \rightarrow$	3	

Drop the cell (2,2) from the table 6 since availability of source S_2 is less than the requirement of destination D_2 . Finally, there is only single cost 3 in the cell(1,2), so allocate in the cell (1,2) $i.e.\ x_{12} = 1$. Thus, requirements of all destinations are fulfilled and the variables at level 1 are $x_{31}, x_{12}, x_{23}, x_{44}$ and x_{15} . Finally, the total cost of bulk transportation is $C = 1 + 3 + 1 + 3 + 1 = 9$.

5 Comparative Study

A comparative study is done on the optimal solution and optimal cost by both the existing methods [13], 14] and the proposed method and it is seen that the proposed algorithm provides the optimal solution in just three steps vis-a-vis five steps by the two methods, apart from being very simple to apply. Thus, approximately 50% reduction in the number of steps is seen when the proposed algorithm is applied to obtain the optimal solution of the bulk transportation problem vis-a-vis the other two methods.

The comparative study is shown in Table 7.

Table 7(Comparative Study)

Parameters	Iterative Method of Maio and Roveda [13]	Branch and Bound Method of Srinivasan and Thompson [14]	Proposed Heuristic Method
Optimal solution Vector	$\{x_{31}, x_{12}, x_{23}, x_{44}, x_{15}\}$	$\{x_{31}, x_{12}, x_{23}, x_{44}, x_{15}\}$	$\{x_{31}, x_{12}, x_{23}, x_{44}, x_{15}\}$
Optimal Cost	9	9	9
Number of steps	5	5	3

6 Concluding Remarks

In the cost minimizing bulk transportation problem the proposed algorithm is seen to give the optimal solution vector (corresponding to the optimal cost) in almost half the number of steps than the existing methods. It needs to be emphasized here that each step of the proposed algorithm is easy to apply and takes less time as compared to the other two methods. Thus, the proposed algorithm is very simple to use and is less time consuming as compared to the existing methods, making it a quick and handy tool to calculate the optimum cost in bulk transportations.

References

[1] Hitchcock, F.L., “The distribution of a product from several sources to numerous locations”, *Journal of Mathematics and Physics*, 20(1941), 224-230.
[2] Dantzig, G.B., “Linear Programming and Extensions”, *Princeton University Press, Princeton, New Jersey*, (1963).
[3] Balas, E., “An additive algorithm for solving linear programs with zero-one variables”, Operations Research, 13(1965), 517-545.
[4] Bhatia, H.L., Puri, M.C., and Swarup. K., “A Procedure for Time Minimizing Transportation Problem”, Indian Journal of Pure & Applied Mathematics, 8(1977), 920-929.
[5] Bhatia, H.L., Puri, M.C., and Swarup, K., “Time-cost trade-off in a transportation problem”, Opsearch,13(1976), 129-142.
[6] Garfinkel, R.S., and Rao, M.R., “The Bottleneck Transportation Problem”, Naval Research Logistics Quarterly, 18(1971), 465-472.
[7] Hammer, P.L., “Time Minimizing Transportation Problems”, Naval Research Logistics Quarterly,16(1970), 345-357.
[8] Hammer, P.L., “Time Minimizing Transportation Problems”, Naval Research Logistics Quarterly, 18(1971), 487-490.
[9] Ramakrishnan, C.S., “A Note on Time Minimizing Transportation Problem”, Opsearch,14(1977), 207- 209.
[10] Seshan, C.R., and Tikekar, V.G., “On Sharma-Swarup algorithm for time minimizing transportation problems”, Proceedings of Indian Academy of Mathematical Sciences, 89(1980),101-102.
[11] Sharma, J.K., and Swarup, K., “Time minimizing transportation problem", Proceedings of Indian Academy of Sciences-Mathematical Sciences, 86(1977), 513-518.
[12] Szwarc, W., “Some Remarks on the Time minimizing Transportation Problems”, Naval Research Logistics Quarterly, 18(1971), 473-485.

[13] Maio, A.D., and Roveda, C., "An all zero-one algorithm for a certain class of transportation problems", Operations Research, 19(1971), I406-1418.
[14] Srinivasan, V., and Thompson, G.L., "An algorithm for assigning uses to sources in a special class of transportation problems", Operations Research, 21(1973), 284-295.
[15] Murthy, M.S., "A bulk transportation problem", Opsearch, 13(1976), I43-155.
[16] Bhatia, H.L., "A Note on a zero-one time minimizing transportation problem", NZOR, 7,(1979), 159-165.
[17] Foulds, L.R., and Gibbons, P.B., "New algorithms for the Bulk Zero-One time mini-max transportation model", NZOR, 8(1980), 109-119.
[18] Verma, V., and Puri, M.C., "A branch and bound method for cost minimizing Bulk transportation problem", Opsearch, 33(1996), 145-161.